

Problem 4. Let X be a connected, closed oriented 3-manifold with fundamental group $\pi_1(X, x_0)$. Using the universal coefficient theorem and Poincaré duality, compute the integral homology and cohomology groups of X in terms of $\pi_1(X, x_0)$. Use this to explain your answer to Problem 3.

Hint Recall that $H_1(X) \cong \pi_1(X, x_0)^{\text{ab}}$, the abelianization of the fundamental group. Express your answer in terms of F and T , which denote the free and torsion parts of $\pi_1(X, x_0)^{\text{ab}}$.