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Research Statement

The primary focus of my current research program is four-dimensional differential topology and non-abelian gauge theory. For my thesis project, I studied the **Vafa-Witten equations** (1.2.1) over a general oriented Riemannian four-manifold. My eventual goal is **to use these equations to establish new invariants of smooth four-manifolds**, analogous to the tremendously successful invariants of Donaldson and Seiberg-Witten.

I explain some background material in Section 1, and describe some connections with various fields in math and physics including algebraic geometry, number theory, quantum field theory, and S-duality. Although these invariants currently lack a rigorous mathematical basis, physicists have conjectured that the generating functions for these invariants should be modular forms. Numerous independent computations from algebraic geometry support this conjecture, providing compelling evidence that these invariants are well-defined. As described in 1.4, the Vafa-Witten equations were recently extended by Haydys and Witten to a five-dimensional gauge theory which is being used to study Khovanov homology and Fukaya-Seidel categories [Hay10, Wit11].

In Section 2 I state my specific research goals. I describe how the Vafa-Witten equations present new challenges which are not present in other similar equations. Consequently, several analytical obstacles must be overcome before the Vafa-Witten invariants can be rigorously defined, namely compactness and transversality. As explained in 2.2, the partial compactness result of my thesis is one step towards this aim. Continuing this work, my immediate goal is to **establish a complete compactification of the moduli space,** and to understand under which circumstances the Vafa-Witten equations lead to well-defined four-manifold invariants.

Another topic I plan to investigate is **moduli spaces of *singular* solutions to the Vafa-Witten equations**, which I discuss in 2.3. The theory of singular connections (which I review in 1.3) was developed by Kronheimer and Mrowka [KM93, KM95b] to allow singularities of prescribed holonomy along an embedded surface. Singular connections were first used to understand constraints on the genus of embedded surfaces, and later used to determine the structure of Donaldson invariants over manifolds of "simple type." The singular Vafa-Witten moduli spaces are widely believed to be related to Jacobi forms, but surprisingly little work has been done from the analytical perspective.

1 Background

1.1 The Donaldson invariants

During the 1980s and the early 1990s, the moduli space of anti-self-dual (ASD) instantons was the primary tool for studying smooth four-manifolds. Let (M, g) be an oriented Riemannian four-manifold, and let G be a compact Lie group. Given a principal G-bundle $P \to M$, a connection $A \in \mathcal{A}_P$, has curvature $F_A \in \Omega^2(M; \mathrm{ad}_P)$. The Hodge star operator splits two-forms into metric-dependent ± 1 eigenspaces $\Omega^{2,\pm}(M; \mathrm{ad}_P)$. Correspondingly, curvature decomposes as $F_A = F_A^+ + F_A^-$. For a specific choice of (M, g) and P, we define *ASD moduli space* as

$$\mathcal{M}_{\mathrm{ASD}}^{P,g} := \{A \in \mathcal{A}_P | F_A^+ = 0\} / \mathcal{G}_P,$$

where $\mathcal{G}_P$ is the group of automorphisms of P. When non-empty, $\mathcal{M}_{\mathrm{ASD}}^{P,g}$ typically is a finite-dimensional submanifold of $\mathcal{A}_P/\mathcal{G}_P$. Roughly speaking, Donaldson showed how for a principal bundle P with $G = \mathrm{SU}(2)$, the moduli space $\mathcal{M}_{\mathrm{ASD}}^{P,g} \subset \mathcal{A}_P/\mathcal{G}_P$ determines a homology class. Numerical invariants of 4-manifolds then arise by pairing this homology class with the cohomology classes of $\mathcal{A}_P/\mathcal{G}_P$ [Don90, DK90]. These invariants have many applications including distinguishing smooth structures on four-manifolds, and understanding connected-sum decompositions.

The proof that Donaldson's invariants are well-defined is notorious for the analytic challenges involved. For example, sometimes $\mathcal{M}_{\text{ASD}}^{P,g}$ has singularities and is non-compact. Usually the singularities can be eliminated via metric perturbations, and there is a natural Uhlenbeck compactification $\overline{\mathcal{M}}_{\text{ASD}}^{P,g}$. To define invariants of the underlying smooth structure, one must ensure that the construction is independent of the choice of metric g. Roughly, this amounts to showing that different choices of metric lead to homologous $\overline{\mathcal{M}}_{\text{ASD}}^{P,g}$.

1.2 Supersymmetric Yang-Mills theory, S-duality, and the Vafa-Witten equations

S-duality, also known as Montonen-Olive duality, electric-magnetic duality, or strong-weak duality, is a conjectural duality between quantum field theories (or string theories), which relates respective behaviors at strong and weak coupling. Since the mathematical foundations of quantum field theory have yet to be established, for this subsection we adopt the physicist's level of rigor without further qualification.

The $\mathcal{N}=2$ and $\mathcal{N}=4$ supersymmetric Yang-Mills theories over $\mathbb{R}^4$ can be "twisted" so that when generalized to a Riemannian four-manifold (X, g), the observables become independent of the choice of metric g. Assuming that such observables are well-defined, they should be differential-topological invariants for the smooth structure of X.

In the $\mathcal{N}=2$ case, there is a unique such twist called "Donaldson-Witten theory" [Wit88], and the observables are the Donaldson invariants. The supersymmetric Lagrangian corresponds to a Mathai-Quillen path integral, which computes the characteristic numbers for the Euler class of the gauge-equivariant map $A \mapsto F_A^+$ [AJ90]. Electric-magnetic duality leads to a formula expressing the Donaldson invariants in terms of the Seiberg-Witten invariants, known as Witten's conjecture [Wit94]. Thanks to the $\mathrm{SO}(3)$ monopole program of Feehan and Leness (a.k.a. the $\mathrm{PU}(2)$ monopole program), many cases of Witten's conjecture now have a rigorous proof [FL06].

With $\mathcal{N}=4$ supersymmetry, there are three inequivalent topological twists [Yam88, Mar95]. Once again, each twist has a Mathai-Quillen expression [LL97], so we obtain equations for each twist. The "Vafa-Witten twist" analyzed in [VW94] corresponds to the equations

$$\begin{aligned} F_A^+ - \tfrac{1}{2}[B \times B] + [C, B] &= 0, \\ d_A^* B - d_A C &= 0, \end{aligned} \quad \text{where} \quad \begin{aligned} &A \in \mathcal{A}_P, \\ &B \in \Omega^{2,+}(M; \mathrm{ad}_P), \\ &C \in \Omega^0(M; \mathrm{ad}_P). \end{aligned} \tag{1.2.1}$$

(The product denoted by $\beta_1 \times \beta_2$ for $\beta_i \in \Omega^{2,+}(M;\mathbb{R})$ is pointwise-equivalent to the cross product $\times$ on $\mathbb{R}^3$.)

The virtual dimension of the corresponding moduli space is zero, so for each topological type $[P]$ of principal G-bundle $P \to X$, one expects a single numerical invariant $\mathrm{VW}(X,[P])$ defined as a regularized Euler number.

Under certain restrictions on the Riemannian curvature of X, [VW94] explained how the resulting moduli space $\mathcal{M}_{\text{VW}}^{P,g}$ of solutions to the Vafa-Witten equations coincides with the ASD moduli space $\mathcal{M}_{\text{ASD}}^{P,g}$, and how the Vafa-Witten invariant should be

$$\mathrm{VW}(X,[P]) = \chi\left(\overline{\mathcal{M}}_{\text{ASD}}^{P,g}\right), \tag{1.2.2}$$

where $\overline{\mathcal{M}}_{\text{ASD}}^{P,g}$ denotes some compactification, and χ is the Euler characteristic. Note that while the moduli space $\overline{\mathcal{M}}_{\text{ASD}}^{P,g}$ depends on the metric up to cobordism, the Euler characteristic is not cobordism-invariant.

However, the aforementioned restrictions on Riemannian curvature ensure that the topology of $\overline{\mathcal{M}}^{P,g}_{\mathrm{ASD}}$ cannot change.

For any simple, simply-laced Lie group G, one can assemble the Vafa-Witten invariants for principal G-bundles into a generating function

$$\sum_k \mathrm{VW}(X,k) q^k \tag{1.2.3}$$

according to the instanton number. Vafa and Witten observed that S-duality predicts a modular transformation law, which relates the generating functions corresponding to G and its Langlands dual group $G^\vee$. The full details can be found in [VW94, Wu08].

1.3 Singular connections

Kronheimer originally proposed the use of singular connections as a tool for understanding the genus of embedded surfaces in four-manifolds [Kro91]. One chooses an embedded surface $\Sigma \subset X$ and a parameter $\alpha \in (0, \frac{1}{2})$. A singular $\mathrm{SU}(2)$ connection is then allowed to have holonomy which is conjugate to

$$\begin{pmatrix} e^{2\pi i\alpha} & 0 \\ 0 & e^{-2\pi i\alpha} \end{pmatrix}$$

along any infinitesimal loop around Σ. When applied to the ASD equations, singular connections are a powerful tool because of how they interpolate between an ordinary $\mathrm{SU}(2)$ moduli space as $\alpha \to 0$, and an $\mathrm{SO}(3)$ moduli space as $\alpha \to \frac{1}{2}$. Moreover, this interpolation incorporates the topology of Σ.

Kronheimer's proposed program was successfully carried out by Kronheimer and Mrowka [KM93, KM95b]. Their results also managed to prove an extremely powerful structure theorem for the Donaldson invariants [KM94, KM95a]. Roughly, their structure theorem implies (under certain restrictions) that the infinitely many Donaldson invariants are determined by just a finite collection of data. According to the mostly-proven Witten conjecture, this data turns out to be the Seiberg-Witten invariants.

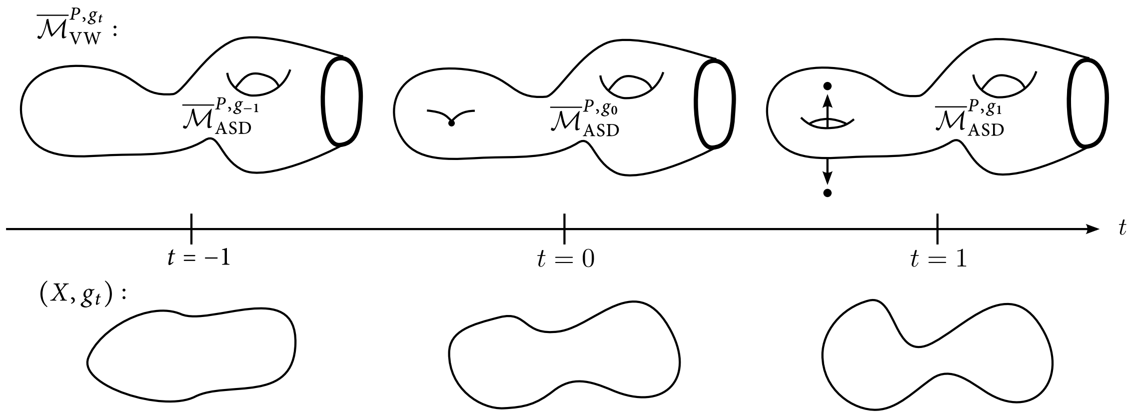
1.4 Recent developments

Since their discovery in 1994, the Seiberg-Witten invariants [Wit94] essentially replaced the role of the far more difficult Donaldson invariants. However the past few years have seen a resurgence of activity involving the ASD equations and other non-abelian gauge theories. In particular, there have been several recent applications of instanton Floer homology and/or singular connections to knot theory, e.g. [KM11, HK10].

The Vafa-Witten equations also recently appeared as a four-dimensional reduction of a particular five-dimensional gauge theory, independently discovered by Haydys [Hay10, (14)] and Witten [Wit11, (5.36)]. According to Haydys, these equations are expected to produce numerical invariants of five-manifolds, Floer homology groups in four dimensions which "categorify" the Vafa-Witten invariants, and a Fukaya-Seidel category in three dimensions [Hay10]. According to Witten, these equations are expected to compute Khovanov homology under appropriate boundary conditions [Wit11]. One of these boundary conditions involves fields which are asymptotic to solutions of the Vafa-Witten equations.

Currently the aforementioned results from this new Haydys-Witten gauge theory are speculative, due in large part to the same analytic difficulties plaguing the Vafa-Witten invariants. A better understanding of the Vafa-Witten equations would certainly help towards understanding the properties of this five-dimensional theory.

Figure 1 – An idealized view of $\overline{\mathcal{M}}_{\mathrm{VW}}$: while deforming the metric g_t on X, the Euler characteristic of $\overline{\mathcal{M}}_{\mathrm{ASD}}$ can change. Any change to $\chi(\overline{\mathcal{M}}_{\mathrm{ASD}})$ should balance with the creation or destruction of points in $\overline{\mathcal{M}}_{\mathrm{VW}} \backslash \overline{\mathcal{M}}_{\mathrm{ASD}}$.



2 Research objective: a mathematical definition of the Vafa-Witten invariants

2.1 The problem

While the conjectural Vafa-Witten invariant $\mathrm{VW}(X, [P])$ has a geometric interpretation under particular circumstances as the Euler characteristic of $\overline{\mathcal{M}}^{P,g}_{\mathrm{ASD}}$, as in (1.2.2), this definition is not viable in general. Different choices of the metric g lead to cobordant moduli spaces, and Euler characteristic is *not* invariant under cobordism! Instead we must look to the full Vafa-Witten equations to formulate a proper definition.

When $\mathcal{M}_{\mathrm{VW}}$ is a union of points cut out transversely by the equations (1.2.1), the Vafa-Witten invariant should be a signed count

$$\mathrm{VW}(X, [P]) = \sum_{x \in \mathcal{M}^{P,g}_{\mathrm{VW}}} (-1)^{\epsilon(x)}.$$

However, $\mathcal{M}_{\mathrm{VW}}$ contains a copy of $\mathcal{M}_{\mathrm{ASD}}$ because if $F^+_A = 0$, then $(A, B, C) = (A, 0, 0)$ is a solution. When $\mathcal{M}_{\mathrm{ASD}}$ has positive dimension, the above summation makes no sense. Instead, consider the complement

$$\widetilde{\mathcal{M}}_{\mathrm{VW}} := \mathcal{M}_{\mathrm{VW}} \backslash \mathcal{M}_{\mathrm{ASD}}.$$

If $\widetilde{\mathcal{M}}_{\mathrm{VW}}(P, g)$ is cut out transversely by finitely many points, then we could try to define

$$\mathrm{VW}(X, [P]) = \chi(\overline{\mathcal{M}}_{\mathrm{ASD}}(P, g)) + \sum_{x \in \widetilde{\mathcal{M}}_{\mathrm{VW}}(P,g)} (-1)^{\epsilon(x)}.$$

This would require proving compactness and transversality for $\widetilde{\mathcal{M}}_{\mathrm{VW}}(P, g)$. Then one must show that the resulting answer is independent of g, as illustrated in Figure 1.

Unfortunately, these assumptions about $\mathcal{M}_{\mathrm{VW}}$ are overly optimistic. For a concrete example, consider the three-dimensional reduction $X = S^1 \times Y$ to some oriented Riemannian three-manifold Y. If the principal bundle P pulls back from Y, then the Vafa-Witten equations reduce to Corlette's equations [Cor88]. When G is semisimple with complexification $G^{\mathbb{C}}$, Corlette proves that the moduli space is the $G^{\mathbb{C}}$ character variety, and B corresponds to the imaginary part of a flat $G^{\mathbb{C}}$-connection.

Since character varieties are affine varieties, often with positive dimension, they can be noncompact. While non-compactness due to bubbling of ASD connections is easily handled via the Uhlenbeck compactification [DK90], the surprising new feature is non-compactness arising from the extra field B. Indeed, compactness properties due to spinor fields are precisely what make the analysis of the Seiberg-Witten equations [KM07] and $\mathrm{PU}(2)$ monopole equations [Tel00, FL98] so tractable. However for the Vafa-Witten equations, the Uhlenbeck compactification is no longer sufficient.

There are two obvious methods of attack on the compactification problem. One possibility would be to find a perturbation from which both compactness and transversality follow. Since the moduli space has virtual dimension zero, it's conceivable that some perturbation can always produce a transverse and finite moduli space. However, perturbations seem to hurt a priori estimates, making compactness more difficult.

A more promising approach is to examine the Morgan-Shalen compactification of the character variety [MS84, MS85]. If I can find the appropriate four-dimensional generalization, the same mechanism may lead to a useful compactification of the Vafa-Witten moduli space.

2.2 Analytic progress

The first step toward proving a compactness theorem for the ASD, Seiberg-Witten, or $\mathrm{PU}(2)$ monopole equations is to obtain an a priori L^2_1 bound for all solutions. This results from expanding and rearranging the L^2 norm of the equations. Proceeding in the same way for the Vafa-Witten equations, one computes

$$\underbrace{\left\|F_A^+ - \tfrac{1}{2}[B\times B] + [C,B]\right\|^2 + \|d_A^* B - d_A C\|^2}_{\text{non-negative, and zero for solutions}} = \underbrace{\tfrac{1}{12}\int_X \langle B\cdot(s-6W^+)B\rangle}_{\text{Riemannian curvature term}} +$$
$$+ \underbrace{\tfrac{1}{2}\|F_A\|^2 + \tfrac{1}{4}\|\nabla_A B\|^2 + \tfrac{1}{2}\|d_A^* B\|^2 + \|d_A C\|^2 + \|[C,B]\|^2 + \tfrac{1}{4}\|[B\times B]\|^2}_{\text{essentially equivalent to the } L^2_1 \text{ norm of } (A,B,C)} + \underbrace{\tfrac{1}{2}\int_X \langle F_A \wedge F_A\rangle}_{\text{topological}}. \quad (2.2.1)$$

Here s is scalar curvature, and W^+ is the self-dual Weyl tensor, which acts on self-dual two-forms as a traceless symmetric endomorphism [AHS78]. We wish to bound the "L^2_1" terms for solutions. The left hand side vanishes for solutions. Since the "topological" term is constant for any fixed principal bundle, the only obstacle is the "Riemannian curvature term," which is quadratic in B. If the quartic term $\frac{1}{4}\|[B\times B]\|^2$ were positive definite, it would dominate the Riemann term, allowing us to control the Riemannian term by completing the square. Positive-definiteness of the quartic term occurs for the Seiberg-Witten equations and the $\mathrm{PU}(2)$ monopole equations, but not for the Vafa-Witten equations. Analytically, this the reason the Vafa-Witten moduli space fails to be Uhlenbeck-compact.

In my thesis, I showed that compactness fails only in a rather tame way. In particular, I use a mean-value inequality to show that a sequence can fail to be compact only when the L^2 norm of B approaches infinity:

Theorem 1 ([Mar10, Theorem 3.5.2]). *Let $P\to X$ be a principal $\mathrm{SU}(2)$ bundle over an oriented Riemannian four-manifold X. For any constant b, the Uhlenbeck closure of $\left\{(A,B,0)\in\mathcal{M}_{\mathrm{VW}}^{P,g} \text{ s.t. } \|B\|_{L^2}\le b\right\}$ is compact.*

2.3 Singular solutions to the Vafa-Witten equations

Recall from (1.2.3) that when we assemble the Vafa-Witten invariants for each instanton number into a generating function, S-duality predicts that this generating function transforms as a modular form. For singular instantons, there is an additional topological number: the monopole number. The generating function for singular instantons is therefore a function of two formal parameters. According to the work of [MNVW98, Yos99, Kap00], S-duality predicts that this generating function transforms as a Jacobi form.

When X is Kähler, there is a Kobayashi-Hitchin correspondence between parabolic bundles and singular ASD connections. This led to some curious algebro-geometric computations of Jacobi forms in the aforementioned references. These computations are carried out as in (1.2.2), under the assumption that the Vafa-Witten invariants are calculable from the moduli space of parabolic bundles. By keeping track of boundary terms in (2.2.1), it should be straightforward to determine under which conditions this assumption is justifiable.

Finally, on a more speculative note, it would be interesting to investigate the singular Vafa-Witten moduli spaces and understand the implications. In analogy with the program of Kronheimer and Mrowka, one could try to understand the interplay between embedded surfaces and the structure of Vafa-Witten invariants.

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