$$\begin{array}{c} \mathbb{C}^? \to E \\ \downarrow \\ M^n \end{array}$$

~~[illegible]~~ Fix $A \in C^\infty$ connection in $\mathcal{A}^{n/2\,*}_1$,

$$S_A(\varepsilon) = \{A + a \mid d_A^* a = 0,\ \|a\|_{L^n} < \varepsilon\} \subset \mathcal{A}^{n/2\,*}_1 .$$

$$m : S_A(\varepsilon) \times \mathcal{G}^{n/2\,*}_2 / \mathrm{Stab}(A) \longrightarrow \mathcal{A}^{n/2\,*}_1$$

We know that this is injective with injective differential. (Diffeo onto its image)

We want to see that image contains an L^n-ball about A

$\mathcal{G}^n_1$ acts

dim $n = 4$, $L^2_1 \hookrightarrow L^4$.

L^n-ball of radius $\delta = U_\delta$

$$m(S_A(\varepsilon) \times \mathcal{G}^{n/2\,*}_2) \cap U_\delta = W_\delta$$

To show: For δ small enough, $U_\delta = W_\delta$.

Certainly $W_\delta \subset U_\delta$. U_δ is connected open. We'll show that W_δ is open and closed in U_δ.

W_δ is the intersection of open sets in the $L^{n/2\,+}_1$-topology, hence open.
W_δ is closed in the $L^{n/2\,+}_1$ topology, i.e. . . .

i.e. $A_i = A + b_i$ sequence of connections in W_δ

$\Rightarrow \exists\ u_i \in \mathcal{G}^{\frac{n}{2}+}_{2},\ a_i \in L^{\frac{n}{2}+}_{1},\ u_i \cdot (A+a_i) = b_i,$
$A + a_i \in S_A(\epsilon).$

$b_i \xrightarrow{L^{\frac{n}{2}+}_{1}} b$ to show u_i, a_i can be chosen to converge in $(L^{\frac{n}{2}+}_{2}, L^{\frac{n}{2}+}_{1})$ topology ~~Sta~~ to $u_i \cdot a$, $\|a\|_{L^n}$ small.

$u \cdot (A + a_i) = A + b_i,\quad d_A^* a_i = 0$

$d_A u_i = -a_i u_i + u_i b_i$

We have a closed decomposition $L^p_k(X; \Lambda^1 \otimes \mathrm{ad}\, P)$
$= d_A(L^p_{k+1}(X, \mathrm{ad}\, P)) + \ker(d_A^* : L^p_k(X; \Lambda^1 \otimes \mathrm{ad}\, P) \to L^p_{k-1}(X; \mathrm{ad}\, P))$

Given $\theta \in L^p_k(X, \Lambda^1 \otimes \mathrm{ad}\, P)$, $\theta = \underset{\mathrm{Im}\, d_A}{\underset{\in}{\theta^e}} + \underset{\ker d_A^*}{\underset{\in}{\theta^c}}$

<u>Lemma 1:</u> $\|(ua)^e\|_{L^n}, \|(au)^e\|_{L^n} \le C \|du\|_{L^n} \|a\|_{L^n}$

<u>Lemma 2:</u> $\|(ua)^e\|_{L^{\frac{n}{2}+}_{1}}, \|(au)^e\|_{L^{\frac{n}{2}+}_{1}} \le C \|du\|_{L^{\frac{n}{2}+}_{1}} \|a\|_{L^n}$

$(ua)^e = \underbrace{d_A (d_A^* d_A)^{-1}}_{L^p_k \to L^p_{k+1}} d_A^*(ua)$ In case 1 we need to estimate the $\|d_A^*(ua)\|_{L^n_{-1}}$.

$d_A^*(ua) = -* d_A * (ua) = -* d_A (u * a) = -* d_A u * a - * u (\cancel{d_A * a})$ $\to 0$.

$\|du \cdot a\|_{L^n_{-1}} \le \|du\|_{L^n} \|a\|_{L^n}$. (NTS $L^n \times L^n \to L^n_{-1}$. $L^{\frac{n}{n-1}}_{1} \to L^{\frac{n}{n-2}}$
$L^n \times L^n \times L^{\frac{n}{n-1}}_{1} \to L^1$.

Lemma: $\|(ua)^e\|_{L^{n/2+}_1} \leq C\|d_A^*(ua)\|_{L^{n/2+}}$

$\leq C\|du \cdot a\|_{L^{n/2+}} \leq C\|du\|_{L^{n/2+}_1}\|a\|_{L^n}$

$L^{n/2+}_1 \times L^n \xrightarrow{?} L^{n/2+}$ (Here we need + small)

Lemma: $\exists\, K, \delta_1 > 0$ so that if $A+a \in S_A(\delta_1)$, $u \in \mathcal{G}^{n/2+}_2$, $u(A+a) = A+b$

$A+b \in \mathcal{A}^{n/2+}_1$ ~~and $\|a\|_{L^n} + \|b\|_{L^n} \leq$~~ $\Longrightarrow \|d_A u\|_{L^n} + \|a\|_{L^n} \leq K\|b\|_{L^n}$.

Proof: $\|d_A u\|_{L^n} + \|a\|_{L^n} = \|d_A u\|_{L^n} + \|ua\|_{L^n}$ (since $|u| = 1$ a.e.)

$\leq \|d_A u\|_{L^n} + \|(ua)^c\|_{L^n} + \|(ua)^e\|_{L^n}$ using closed decomp $L^n = d_A L^n_1 \oplus \ker d_A^*$.

$\leq C\|d_A u + (ua)^c\|_{L^n} + \|(ua)^e\|_{L^n}$

$\leq C\|d_A u + ua\|_{L^n} + (C+1)\|(ua)^e\|_{L^n}$ ~~$d_A^* d_A u = 0 \Rightarrow d_A u = 0$~~

$\leq C\|bu\|_{L^n} + (C+1)\|(ua)^e\|_{L^n}$

$\leq C\|b\|_{L^n} + (C+1)C'\|d_A u\|_{L^n}\|a\|_{L^n}$

If $(C+1)C'\|a\|_{L^n} \leq \frac{1}{2}$ say $\Longrightarrow \frac{1}{2}\|d_A u\|_{L^n} + \|a\|_{L^n} \leq C\|b\|_{L^n}$.

Now $L^{n/2+}_1$. ~~At~~ ~~Replace~~ we ~~try~~

$\|d_A u\|_{L^{n/2+}_1} + \|a\|_{L^{n/2+}_1} \leq \|d_A u\|_{L^{n/2+}_1} + \|\nabla a\|_{L^{n/2+}} + \|a\|_{L^{n/2+}}$

$= \|d_A u\|_{L^{n/2+}_1} + \|u\nabla a\|_{L^{n/2+}} + \|ua\|_{L^{n/2+}}$

$\leq \|d_A u\|_{L^{n/2+}_1} + \|\nabla(ua)\|_{L^{n/2+}} + \|ua\|_{L^{n/2+}} + \|\nabla u \cdot a\|_{L^{n/2+}}$

$\leq \|d_A u\|_{L^{n/2+}_1} + \|ua\|_{L^{n/2+}_1} + \|\nabla u \cdot a\|_{L^{n/2+}}$

$\leq \|d_A u\|_{L^{n/2+}_1} + \|(ua)^c\|_{L^{n/2+}_1} + \|(ua)^e\|_{L^{n/2+}_1} + \|du\|_{L^{n/2+}_1}\|a\|_{L^n}$

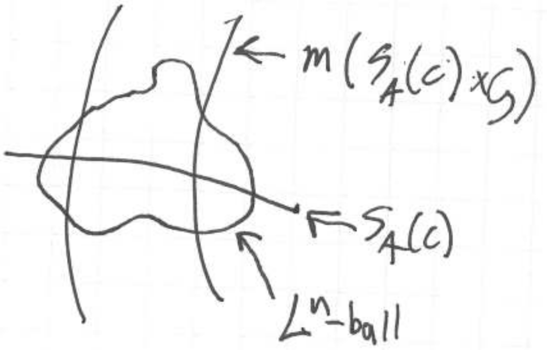
$$\le C\left(\|d_A u + (ua)^c\|_{L^{n/2+}_1}\right) + \|(ua)^c\|_{L^{n/2+}_1} + \|du\|_{L^{n/2+}_1}\|a\|_{L^n}$$

$$\le C\|d_A u + ua\|_{L^{n/2+}_1} + (C+1)\|(ua)^*\|_{L^{n/2+}_1} + \|du\|_{L^{n/2+}_1}\|a\|_{L^n}$$

$$\le C\left(\|b\|_{L^{n/2+}_1} + \|du\|_{L^{n/2+}_1}\|b\|_{L^n}\right) + (C+1)C'\|du\|_{L^{n/2+}_1}\|a\|_{L^n} + \|du\|_{L^{n/2+}_1}\|a\|_{L^n}$$

Thus we have the Lemma: $\exists K, \delta$ s.t. if $b \in \mathcal{A}^{n/2+}_1$, $u \in \mathcal{G}^{n/2+}_2$, $a \in S_A(\varepsilon)$, $u \cdot (A+a) = A+b$ and $\|a\|_{L^n} + \|b\|_{L^n} < \delta$,

$$\|d_A u\|_{L^{n/2+}_1} + \|a\|_{L^{n/2+}_1} \le K\|b\|_{L^{n/2+}_1}.$$



$A + b_i \xrightarrow{L^{n/2+}_1} A + b$ $\quad \|b_i\|_{L^n} \le \delta_1$

$\exists u_i \in \mathcal{G}^{n/2+}_1$, $u_i \subset S_A(c)$

$u_i(A + a_i) = A + b_i$

Taking δ_1 small enough so that $K\delta_1 < \varepsilon \Rightarrow c_i$